PORTFOLIO MANAGEMENT
CLASS 4

→ weights kya use kare jisse sabse kam risk aaye

Minimum Risk Portfolio (2 stock world)

Two stocks A & B
sd 12% 8%

$r_{AB} = 0.1$

$\therefore Cov(A,B) = r \times sd \times sd$
$= 12 \times 8 \times 0.1$
$= 9.6$

A & B combine
⇓
How many portfolios are possible?
↓
Ans: Infinite

Unsaare portfolios ka alag alag s.d (σp) aayega

Ex: $W_A = 50\%$ $W_B = 50\%$
Calculate σp.

ko sochega ro dekhega 8 & 12 ka σp k beech wala aana chahie but because of diversification risk sabhi kam aya hai i.e. risk kata hai

$\therefore \sigma p = \sqrt{(.5 \times 12)^2 + (.5 \times 8)^2 + 2 \times .5 \times .5 \times 9.6}$
$= \sqrt{56.8}$
$= 7.54\%$

Ex: $W_A = 30\%$ $W_B = 70\%$
calculate σp.

$\therefore \sigma p = \sqrt{(.3 \times 12)^2 + (.7 \times 8)^2 + 2 \times .3 \times .7 \times 9.6}$
$= \sqrt{12.96 + 31.36 + 4.032}$
$= 6.95\%$

Construct Minimum Risk portfolio?
⇓
Find out W_A & W_B such that σp is least

Formula :-

$$\boxed{W_A = \frac{\text{Doosre ka variance} - \text{covariance}}{\text{Dono ka variance} - 2\,\text{covariance}}}$$

→ Formula derived from Differential Calculus

$$\therefore W_A = \frac{\sigma B^2 - Cov(A,B)}{\sigma A^2 + \sigma B^2 - 2\,cov(A,B)}$$

$$\therefore W_A = \frac{8^2 - 9.6}{12^2 + 8^2 - 2 \times 9.6} = \frac{64 - 9.6}{144 + 64 - 19.2} = \frac{54.4}{188.8}$$

$= 0.2881$
$\cong 0.29$
$\cong 29\%$ approx

$$\boxed{\therefore W_B = 1 - W_A}$$

$= 71\%$ approx

- If more than 2 stocks ⇒ Quadratic Programming
⇓
Goal seek or solver function

CASE STUDY 1

P	X (ABC)	Y (XYZ)	$P(X-\bar{X})^2$	$P(Y-\bar{Y})^2$	$P(X-\bar{X})(Y-\bar{Y})$
.20	12	16	0.0605	3.042	-0.429
.25	14	10	0.5256	1.1025	-0.71
.25	-7	28	95.55	63.2025	-77.71
.30	28	-2	71.610	59.643	-65.35
			167.74	126.99	-144.253

cov(x, y)

1. $E(R) = 12.55\%$ (b)

2. $E(R)$ for xyz $= 12.1\%$.

3. $\sigma x = \sqrt{167.74}$
$= 12.95$

4. $\sigma y = \sqrt{126.99}$
$= 11.27\%$ (d)

5. 50%.

100%

$E(R_p) = \frac{12.55 + 12.1}{2} = 12.325\%$ (b)

6. $\sigma p = \sqrt{(.5 \times 12.95)^2 + (.5 \times 11.27)^2 + 2 \times .5 \times .5 \times -144.253}$

$= \sqrt{1.5521}$
$= 1.2458$
$\simeq 1.25\%$ (b)

P X Y PX PY $P(X-\bar{X})^2$ $P(Y-\bar{Y})^2$ $(X-\bar{X})$ $(Y-\bar{Y})$ $P(X-\bar{X})(Y-\bar{Y})$

Q6. $W_x = \dfrac{\sigma_y^2 - Cov(x,y)}{\sigma_x^2 + \sigma_y^2 - 2\,Cov(x,y)}$

$= \dfrac{126.99 - (-144.2535)}{167.74 + 126.99 - 2(-144.2535)}$

$= \dfrac{271.2435}{583.237} = 0.47$

$\therefore W_x = 0.465 \rightarrow$ ABC ka weight
$W_y = 0.535 \rightarrow$ XYZ ka weight

Risk free Portfolio (2 stocks)

⇓

only possible when $r = -1$

⇓

$$\boxed{W_A = \dfrac{\text{Doosre ka sd}}{\text{Dono ka sd}}}$$

$W_A = \dfrac{\sigma B}{\sigma A + \sigma B}$

Ex: consider the following 2 stocks which are perfectly negatively correlated :-

CASE STUDY		A	B
	$E(R)$	12%	15%
1.	σ	8%	10%

Q1. Construct a risk free portfolio?

ratio proportion

$W_A = \frac{10}{18} = 0.56$

$W_B = 0.44$

Q2. Calculate $E(R_p)$, σ_p of the portfolio?

$E(R_p) = 12 \times .56 + 15 \times 0.44$
$= 13.32\%$

$\sigma_p = \sqrt{(.56 \times 8)^2 + (.44 \times 10)^2 + 2 \times .56 \times 0.44 \times (-1) \times 8 \times 10}$

$= \sqrt{0.0064}$

$= 0.08$
$\simeq 0$ proved

"N" stock world → It is a bridge between MPT & CAPM.

sums given in RP

Not came in exam till date

$E(R_i) = 12\%$
$\sigma_i = 5\%$
$r = 0.25$

$\therefore$ covariance $= 0.25 \times 5 \times 5 = 6.25$

Equally weighted portfolio of "N" stocks

⇓

$E(R_p) = 12\%$

$$\boxed{\sigma_p = \sqrt{\frac{var - cov}{N} + cov}}$$

$$\sigma_p = \sqrt{\frac{25 - 6.25}{N} + 6.25}$$

$$= \sqrt{\frac{18.75}{N} + 6.25}$$

N = 2 stocks

$$\sigma_p = \sqrt{\frac{18.75}{2} + 6.25}$$

$= 3.95\%$

N = 5 stocks

$$\sigma_p = \sqrt{\frac{18.75}{5} + 6.25}$$

$= 3.16\%$

N = 10 stocks

$$\sigma_p = \sqrt{\frac{18.75}{10} + 6.25}$$

$= 2.85\%$

$N \to \infty$ stocks

$$\sigma_p = \sqrt{\frac{18.75}{N \to \infty} + 6.25}$$

$$= \sqrt{0 + 6.25}$$

$= 2.5\%$ → lowest possible sd ⇒ systematic risk

bcoz it affects economy

unsystematic risk affects internal co.

$E(R_p) = 12\%$

Say $R_f = 4\%$

Sharpe ratio :–

$N = 2 \quad = \dfrac{E(R) - R_f}{\sigma_p} = 2.03$

$N = 5 \quad = 2.53$

$N = 10 \quad = 2.81$

$N \to \infty \quad = 3.2$ ⇒ maximum sharpe ratio

⇓

It is known as the slope of the Capital Allocation Line (CAL)

$N \to \infty$ → This portfolio is known as the Market Portfolio

⇓

Highest Sharpe ratio

So, everyone should invest in the market portfolio "but" combine the same with R_f investment or borrowing.

to enjoy the highest Sharpe ratio.

CAL = CML (Capital Market Line)

$$\boxed{E(R_p) = 4 + 3.2\sigma_p}$$

$$E(R_p) = R_f + \left[\frac{R_m - R_f}{\sigma_m}\right]\sigma$$

$$E(R_p) = R_f + \text{Sharpe ratio} \times \sigma_p$$ → CAL